

19. Many of the operations are done efficiently on most modern graphical calculators using their built-in vector manipulation and rectangular \leftrightarrow polar “shortcuts.” In this solution, we employ the “traditional” methods (such as Eq. 3-6). Where the length unit is not displayed, the unit meter should be understood.

(a) Using unit-vector notation,

$$\begin{aligned}\vec{a} &= 50 \cos(30^\circ) \hat{i} + 50 \sin(30^\circ) \hat{j} \\ \vec{b} &= 50 \cos(195^\circ) \hat{i} + 50 \sin(195^\circ) \hat{j} \\ \vec{c} &= 50 \cos(315^\circ) \hat{i} + 50 \sin(315^\circ) \hat{j} \\ \vec{a} + \vec{b} + \vec{c} &= 30.4 \hat{i} - 23.3 \hat{j} .\end{aligned}$$

The magnitude of this result is $\sqrt{30.4^2 + (-23.3)^2} = 38 \text{ m}$.

- (b) The two possibilities presented by a simple calculation for the angle between the vector described in part (a) and the $+x$ direction are $\tan^{-1}(-23.2/30.4) = -37.5^\circ$, and $180^\circ + (-37.5^\circ) = 142.5^\circ$. The former possibility is the correct answer since the vector is in the fourth quadrant (indicated by the signs of its components). Thus, the angle is -37.5° , which is to say that it is roughly 38° *clockwise* from the $+x$ axis. This is equivalent to 322.5° counterclockwise from $+x$.
- (c) We find $\vec{a} - \vec{b} + \vec{c} = (43.3 - (-48.3) + 35.4) \hat{i} - (25 - (-12.9) + (-35.4)) \hat{j} = 127 \hat{i} + 2.6 \hat{j}$ in unit-vector notation. The magnitude of this result is $\sqrt{127^2 + 2.6^2} \approx 1.3 \times 10^2 \text{ m}$.
- (d) The angle between the vector described in part (c) and the $+x$ axis is $\tan^{-1}(2.6/127) \approx 1^\circ$.
- (e) Using unit-vector notation, \vec{d} is given by

$$\begin{aligned}\vec{d} &= \vec{a} + \vec{b} - \vec{c} \\ &= -40.4 \hat{i} + 47.4 \hat{j}\end{aligned}$$

which has a magnitude of $\sqrt{(-40.4)^2 + 47.4^2} = 62 \text{ m}$.

- (f) The two possibilities presented by a simple calculation for the angle between the vector described in part (e) and the $+x$ axis are $\tan^{-1}(47.4/(-40.4)) = -50^\circ$, and $180^\circ + (-50^\circ) = 130^\circ$. We choose the latter possibility as the correct one since it indicates that \vec{d} is in the second quadrant (indicated by the signs of its components).