

83. (Fifth problem in **Cluster 1**)

A useful diagram (where these forces are analyzed) is Fig. 6-5 in the textbook. In that figure, W is the weight (equal to $mg = 98 \text{ N}$).

- (a) Since there is no motion, then $\sum \vec{F} = 0$ along the incline, so $f_s - W \sin \theta = 0$ (if uphill is positive, which is the direction assumed for \vec{f}_s). We therefore obtain $f_s = 25 \text{ N}$. Our result is positive, so it indeed points uphill as we had assumed. One can check that this value of f_s does not exceed the maximum possible value $f_{s, \max}$ (see next part).
- (b) As in part (a), we have $f_s - W \sin \theta = 0$, but since the system is on the verge of motion we also have $f_s = f_{s, \max} = \mu_s W \cos \theta$. Therefore,

$$\mu_s W \cos \theta - W \sin \theta = 0 \implies \mu_s = \tan \theta$$

which leads to $\theta_s = \tan^{-1} \mu_s = 31^\circ$ (this is often called “the angle of repose”).

- (c) If the block slides with no acceleration then we have $f_k - W \sin \theta = 0$ from Newton’s second law applied along the incline surface. With $f_k = \mu_k W \cos \theta$ we are led to $\theta_k = \tan^{-1} \mu_k$ as the condition for this constant velocity sliding downhill. Since $\mu_k < \mu_s$ then we see that $\theta_k < \theta_s$ from part (b).
- (d) We find $\theta_k = \tan^{-1} \mu_k = 11^\circ$.