

41. The volume of a parallelepiped is equal to the product of its altitude and the area of its base. Take the base to be the parallelogram formed by the vectors \vec{b} and \vec{c} . Its area is $bc \sin \phi$, where ϕ is the angle between \vec{b} and \vec{c} . This is just the magnitude of the vector (cross) product $\vec{b} \times \vec{c}$. The altitude of the parallelepiped is $a \cos \theta$, where θ is the angle between \vec{a} and the normal to the plane of \vec{b} and \vec{c} . Since $\vec{b} \times \vec{c}$ is normal to that plane, θ is the angle between \vec{a} and $\vec{b} \times \vec{c}$. Thus, the volume of the parallelepiped is $V = a|\vec{b} \times \vec{c}| \cos \theta = \vec{a} \cdot (\vec{b} \times \vec{c})$.