

61. The velocity vector (relative to the shore) for ships  $A$  and  $B$  are given by

$$\vec{v}_A = -(v_A \cos 45^\circ) \hat{i} + (v_A \sin 45^\circ) \hat{j}$$

and

$$\vec{v}_B = -(v_B \sin 40^\circ) \hat{i} - (v_B \cos 40^\circ) \hat{j}$$

respectively (where  $v_A = 24$  knots and  $v_B = 28$  knots). We are taking East as  $+\hat{i}$  and North as  $\hat{j}$ .

(a) Their relative velocity is

$$\vec{v}_{AB} = \vec{v}_A - \vec{v}_B = (v_B \sin 40^\circ - v_A \cos 45^\circ) \hat{i} + (v_B \cos 40^\circ + v_A \sin 45^\circ) \hat{j}$$

the magnitude of which is  $|\vec{v}_{AB}| = \sqrt{1.0^2 + 38.4^2} \approx 38$  knots. The angle  $\theta$  which  $\vec{v}_{AB}$  makes with North is given by

$$\theta = \tan^{-1} \left( \frac{v_{AB,x}}{v_{AB,y}} \right) = \tan^{-1} \left( \frac{1.0}{38.4} \right) = 1.5^\circ$$

which is to say that  $\vec{v}_{AB}$  points  $1.5^\circ$  east of north.

(b) Since they started at the same time, their relative velocity describes at what rate the distance between them is increasing. Because the rate is steady, we have

$$t = \frac{|\Delta r_{AB}|}{|\vec{v}_{AB}|} = \frac{160}{38} = 4.2 \text{ h}.$$

(c) The velocity  $\vec{v}_{AB}$  does not change with time in this problem, and  $\vec{r}_{AB}$  is in the same direction as  $\vec{v}_{AB}$  since they started at the same time. Reversing the points of view, we have  $\vec{v}_{AB} = -\vec{v}_{BA}$  so that  $\vec{r}_{AB} = -\vec{r}_{BA}$  (i.e., they are  $180^\circ$  opposite to each other). Hence, we conclude that  $B$  stays at a bearing of  $1.5^\circ$  west of south relative to  $A$  during the journey (neglecting the curvature of Earth).