

33. As hinted in the problem statement, the velocity v of the system as a whole – when the spring reaches the maximum compression x_m – satisfies $m_1 v_{1i} + m_2 v_{2i} = (m_1 + m_2)v$. The change in kinetic energy of the system is therefore

$$\begin{aligned}\Delta K &= \frac{1}{2}(m_1 + m_2)v^2 - \frac{1}{2}m_1 v_{1i}^2 - \frac{1}{2}m_2 v_{2i}^2 \\ &= \frac{(m_1 v_{1i} + m_2 v_{2i})^2}{2(m_1 + m_2)} - \frac{1}{2}m_1 v_{1i}^2 - \frac{1}{2}m_2 v_{2i}^2\end{aligned}$$

which yields $\Delta K = -35$ J. (Although it is not necessary to do so, still it is worth noting that algebraic manipulation of the above expression leads to $|\Delta K| = \frac{1}{2} \left(\frac{m_1 m_2}{m_1 + m_2} \right) v_{\text{rel}}^2$ where $v_{\text{rel}} = v_1 - v_2$). Conservation of energy then requires

$$\frac{1}{2} k x_m^2 = -\Delta K \implies x_m = \sqrt{\frac{-2\Delta K}{k}} = \sqrt{\frac{-2(-35)}{1120}}$$

which gives the result $x_m = 0.25$ m.