

30. Fig. 6-4 in the textbook shows a similar situation (using ϕ for the unknown angle) along with a free-body diagram. We use the same coordinate system as in that figure.

(a) Thus, Newton's second law leads to

$$\begin{aligned} T \cos \phi - f &= ma && \text{along } x \text{ axis} \\ T \sin \phi + N - mg &= 0 && \text{along } y \text{ axis} \end{aligned}$$

Setting $a = 0$ and $f = f_{s,\max} = \mu_s N$, we solve for the mass of the box-and-sand (as a function of angle):

$$m = \frac{T}{g} \left(\sin \phi + \frac{\cos \phi}{\mu_s} \right)$$

which we will solve with calculus techniques (to find the angle ϕ_m corresponding to the maximum mass that can be pulled).

$$\frac{dm}{d\phi} = \frac{T}{g} \left(\cos \phi - \frac{\sin \phi}{\mu_s} \right) = 0$$

This leads to $\tan \phi_m = \mu_s$ which (for $\mu_s = 0.35$) yields $\phi_m = 19^\circ$.

(b) Plugging our value for ϕ_m into the equation we found for the mass of the box-and-sand yields $m = 340$ kg. This corresponds to a weight of $mg = 3.3 \times 10^3$ N.