

67. (a) We use conservation of mechanical energy to find an expression for ω^2 as a function of the angle θ that the chimney makes with the vertical. The potential energy of the chimney is given by $U = Mgh$, where M is its mass and h is the altitude of its center of mass above the ground. When the chimney makes the angle θ with the vertical, $h = (H/2) \cos \theta$. Initially the potential energy is $U_i = Mg(H/2)$ and the kinetic energy is zero. The kinetic energy is $\frac{1}{2}I\omega^2$ when the chimney makes the angle θ with the vertical, where I is its rotational inertia about its bottom edge. Conservation of energy then leads to

$$MgH/2 = Mg(H/2) \cos \theta + \frac{1}{2}I\omega^2 \implies \omega^2 = (MgH/I)(1 - \cos \theta) .$$

The rotational inertia of the chimney about its base is $I = MH^2/3$ (found using Table 11-2(e) with the parallel axis theorem). Thus

$$\omega = \sqrt{\frac{3g}{H}(1 - \cos \theta)} .$$

- (b) The radial component of the acceleration of the chimney top is given by $a_r = H\omega^2$, so $a_r = 3g(1 - \cos \theta)$.
- (c) The tangential component of the acceleration of the chimney top is given by $a_t = H\alpha$, where α is the angular acceleration. We are unable to use Table 11-1 since the acceleration is not uniform. Hence, we differentiate $\omega^2 = (3g/H)(1 - \cos \theta)$ with respect to time, replacing $d\omega/dt$ with α , and $d\theta/dt$ with ω , and obtain

$$\frac{d\omega^2}{dt} = 2\omega\alpha = (3g/H)\omega \sin \theta \implies \alpha = (3g/2H) \sin \theta .$$

Consequently, $a_t = H\alpha = \frac{3g}{2} \sin \theta$.

- (d) The angle θ at which $a_t = g$ is the solution to $\frac{3g}{2} \sin \theta = g$. Thus, $\sin \theta = 2/3$ and we obtain $\theta = 41.8^\circ$.