

45. We observe that $\left| \hat{i} \times \hat{i} \right| = \left| \hat{i} \right| \left| \hat{i} \right| \sin 0^\circ$ vanishes because $\sin 0^\circ = 0$. Similarly, $\hat{j} \times \hat{j} = \hat{k} \times \hat{k} = 0$. When the unit vectors are perpendicular, we have to do a little more work to show the cross product results. First, the magnitude of the vector $\hat{i} \times \hat{j}$ is

$$\left| \hat{i} \times \hat{j} \right| = \left| \hat{i} \right| \left| \hat{j} \right| \sin 90^\circ$$

which equals 1 because $\sin 90^\circ = 1$ and these are all unit vectors (each has magnitude equal to 1). This is consistent with the claim that $\hat{i} \times \hat{j} = \hat{k}$ since the magnitude of \hat{k} is certainly 1. Now, we use the right-hand rule to show that $\hat{i} \times \hat{j}$ is in the positive z direction. Thus $\hat{i} \times \hat{j}$ has the same magnitude and direction as \hat{k} , so it is equal to \hat{k} . Similarly, $\hat{k} \times \hat{i} = \hat{j}$ and $\hat{j} \times \hat{k} = \hat{i}$. If, however, the coordinate system is left-handed, we replace $\hat{k} \rightarrow -\hat{k}$ in the work we have shown above and get

$$\hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k} = 0 .$$

just as before. But the relations that are different are

$$\hat{i} \times \hat{j} = -\hat{k} \quad \hat{k} \times \hat{i} = -\hat{j} \quad \hat{j} \times \hat{k} = -\hat{i} .$$