

41. We adopt the positive direction choices used in the textbook so that equations such as Eq. 4-22 are directly applicable. The coordinate origin is at the point where the ball is kicked. Where units are not displayed, SI units are understood. We use  $x$  and  $y$  to denote the coordinates of ball at the goalpost, and try to find the kicking angle(s)  $\theta_0$  so that  $y = 3.44$  m when  $x = 50$  m. Writing the kinematic equations for projectile motion:  $x = v_0 t \cos \theta_0$  and  $y = v_0 t \sin \theta_0 - \frac{1}{2}gt^2$ , we see the first equation gives  $t = x/v_0 \cos \theta_0$ , and when this is substituted into the second the result is

$$y = x \tan \theta_0 - \frac{gx^2}{2v_0^2 \cos^2 \theta_0}.$$

One may solve this by trial and error: systematically trying values of  $\theta_0$  until you find the two that satisfy the equation. A little manipulation, however, will give an algebraic solution:

Using the trigonometric identity  $1/\cos^2 \theta_0 = 1 + \tan^2 \theta_0$ , we obtain

$$\frac{1}{2} \frac{gx^2}{v_0^2} \tan^2 \theta_0 - x \tan \theta_0 + y + \frac{1}{2} \frac{gx^2}{v_0^2} = 0$$

which is a second-order equation for  $\tan \theta_0$ . To simplify writing the solution, we denote  $c = \frac{1}{2}gx^2/v_0^2 = \frac{1}{2}(9.80)(50)^2/(25)^2 = 19.6$  m. Then the second-order equation becomes  $c \tan^2 \theta_0 - x \tan \theta_0 + y + c = 0$ . Using the quadratic formula, we obtain its solution(s).

$$\begin{aligned} \tan \theta_0 &= \frac{x \pm \sqrt{x^2 + 4(y+c)c}}{2c} \\ &= \frac{50 \pm \sqrt{50^2 - 4(3.44 + 19.6)(19.6)}}{2(19.6)}. \end{aligned}$$

The two solutions are given by  $\tan \theta_0 = 1.95$  and  $\tan \theta_0 = 0.605$ . The corresponding (first-quadrant) angles are  $\theta_0 = 63^\circ$  and  $\theta_0 = 31^\circ$ . If kicked at any angle between these two, the ball will travel above the cross bar on the goalposts.