

35. We establish a coordinate system with the origin at the position of initial nucleus of mass  $m_{mi}$  (which was stationary), with the electron momentum  $\vec{p}_e$  in the  $-x$  direction and the neutrino momentum  $\vec{p}_\nu$  in the  $-y$  direction. We will use unit-vector notation, although the problem does not specifically request it.

- (a) We find the momentum  $\vec{p}_{nr}$  of the residual nucleus from momentum conservation.

$$\begin{aligned}\vec{p}_{ni} &= \vec{p}_e + \vec{p}_\nu + \vec{p}_{nr} \\ 0 &= -1.2 \times 10^{-22} \hat{i} - 6.4 \times 10^{-23} \hat{j} + \vec{p}_{nr}\end{aligned}$$

Thus,  $\vec{p}_{nr} = 1.2 \times 10^{-22} \hat{i} + 6.4 \times 10^{-23} \hat{j}$  in SI units (kg·m/s). Its magnitude is

$$|\vec{p}_{nr}| = \sqrt{(1.2 \times 10^{-22})^2 + (6.4 \times 10^{-23})^2} = 1.4 \times 10^{-22} \text{ kg}\cdot\text{m/s} .$$

- (b) The angle measured from the  $+x$  axis to  $\vec{p}_{nr}$  is

$$\theta = \tan^{-1} \left( \frac{6.4 \times 10^{-23}}{1.2 \times 10^{-22}} \right) = 28^\circ .$$

Therefore, the angle between  $\vec{p}_e$  (which is in the  $-x$  direction) and  $\vec{p}_{nr}$  is  $180^\circ - 28^\circ \approx 150^\circ$ .

- (c) Measuring clockwise (but not using the “traditional” minus sign with that sense) we find the angle between  $\vec{p}_{nr}$  and  $\vec{p}_\nu$  (which points in the  $-y$  direction) is  $90^\circ + 28^\circ \approx 120^\circ$ .
- (d) Combining the two equations  $p = mv$  and  $K = \frac{1}{2}mv^2$ , we obtain (with  $p = p_{nr}$  and  $m = m_{nr}$ )

$$K = \frac{p^2}{2m} = \frac{(1.4 \times 10^{-22})^2}{2(5.8 \times 10^{-26})} = 1.6 \times 10^{-19} \text{ J} .$$