

20. (a) At Q the block (which is in circular motion at that point) experiences a centripetal acceleration v^2/R leftward. We find v^2 from energy conservation:

$$\begin{aligned} K_P + U_P &= K_Q + U_Q \\ 0 + mgh &= \frac{1}{2}mv^2 + mgR \end{aligned}$$

Using the fact (mentioned in problem 6) that $h = 5R$, we find $mv^2 = 8mgR$. Thus, the horizontal component of the net force on the block at Q is $mv^2/R = 8mg$ and points left (in the same direction as \vec{a}).

- (b) The downward component of the net force on the block at Q is the force of gravity mg downward.
(c) To barely make the top of the loop, the centripetal force there must equal the force of gravity:

$$\frac{mv_t^2}{R} = mg \implies mv_t^2 = mgR$$

This requires a different value of h than was used above.

$$\begin{aligned} K_P + U_P &= K_t + U_t \\ 0 + mgh &= \frac{1}{2}mv_t^2 + mgh_t \\ mgh &= \frac{1}{2}(mgR) + mg(2R) \end{aligned}$$

Consequently, $h = 2.5R$.

- (d) The normal force N , for speeds v_t greater than \sqrt{gR} (which are the only possibilities for non-zero N – see the solution in the previous part), obeys

$$N = \frac{mv_t^2}{R} - mg$$

from Newton's second law. Since v_t^2 is related to h by energy conservation

$$K_P + U_P = K_t + U_t \implies gh = \frac{1}{2}v_t^2 + 2gR$$

then the normal force, as a function for h (so long as $h \geq 2.5R$ – see solution in previous part), becomes

$$N = \frac{2mg}{R}h - 5mg.$$

Thus, the graph for $h \geq 2.5R$ consists of a straight line of positive slope $2mg/R$ (which can be set to some convenient values for graphing purposes). For $h \leq 2.5R$, the normal force is zero. In the interest of saving space, we do not show the graph here.